Spontaneous emission of a small source of light.

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The English of this paper must be corrected.

Abstract

The usual computation of the spontaneous emission uses a mixture of classical and quantum postulates. A purely classical computation shows that a source of electromagnetic field absorbs light in the eigenmode it is able to emit. Thus in an excitation by an other mode, the component of this mode on the eigenmode is absorbed, while the remainder is scattered. This loss of energy does not apply to the zero point field which has its regular energy in the eigenmode, so that the zero point field seems more effective than the other fields for the stimulation of light emission.

1 Introduction.

The spontaneous emission may be considered as an amplification of the zero point field [1]. However it seems that the zero point field is twice more effective than the ordinary fields [2, 3, 4].

The starting point of this conclusion is the *classical* computation of the rate at which an oscillating dipole absorbs energy from a plane wave through its electric field, and diffracts a fraction of this energy [5].

The radiation reaction provides the energy necessary to double the spontaneous excitation of a source due to the zero point field; but this computation fails because in the amplification of a beam of light (hot, i.e. $T > 0$ in Planck's law) the zero point field is not a data of the problem.

The origin of the radiation reaction is the interaction between the charges and the electromagnetic field they produce, leading to Abraham-Lorentz [6] or Ford-O'Connell [7] formula. The computations require a model of source, including implicit or explicit Poincaré stresses.

Avoiding the choice of a model of source, using only Maxwell's equations in the vacuum thanks to Schwarzschild-Fokker's trick, reduces the precision of a theory, but allows to fix the limits of all possible computations: It will be

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supposed only that a small source made of charges radiates an electromagnetic field which obeys Maxwell's equations.

Section 2 recalls the theory of the modes of an electromagnetic field in the vacuum.

Section 3 shows how the Schwarzschild-Fokker's trick allows to use the modes with sources.

Section 4 shows that a source can absorb the whole energy of the eigenmode it is able to emit, while its excitation by an other mode requires a projection on the eigenmode which reduces the excitation.

The following classical computation starts the demonstration at the beginning, Maxwell's equations. The electromagnetic field will be generally simply named "field", and represented by its electric field, the magnetic part being implied.

2 Mathematical modes of the electromagnetic field in the vacuum.

In the vacuum, without sources, Helmoltz's equation for the electric field E, which may be deduced from Maxwell's equations reduces to:

$$\nabla^{\mathbf{2}}\mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \tag{1}$$

In the whole space, or in a region limited by perfectly conducting walls, this equation is linear, so that its solutions named "modes" build a vector space.

As there is nether sources, nor loss of energy, the energy W of an electromagnetic field may be computed at any time by the following integration:

$$W = (1/2)\int[\epsilon_0\mathbf{E}^2 + \mu_0\mathbf{H}^2]dV \tag{2}$$

In Physics, the result must be finite, so that plane waves are necessarily approximations.

The energy of the sum of two modes is generally not the sum of the energies of each; else, the modes are orthogonal. To avoid confusions, the energy of a mode supposed alone in the universe will be named "energy of the bare mode" or "bare energy"; next section will show that bare energies do not exist in physics.

If the energy of a bare mode is finite, the mode may be normalised multiplying $\mathbf{E}$ and $\mathbf{H}$ by a constant such that, if $e(\nu)\mathrm{d}\nu$ is the energy of the mode between ν and $\nu + \mathrm{d}\nu$,

$$\int \frac{e(\nu)d\nu}{h\nu} = 1. \tag{3}$$

Thus orthonormal, or at least orthogonal reference frames may be built in the space of the modes.

Remark that introducing a new electromagnetic field into an existing electromagnetic field does not necessarily increase the energy; in particular, introducing a field exactly opposite to an existing field cancels all, so that the energy falls to zero, it is the absorption of the initial field.

3 Schwarzschild-Fokker's trick.

These authors replace charges by convenient fields which provide the same field without the charges.

A source supposed invariant by a time inversion (for instance a temporary dipole) radiates an emission field named "retardated eigenfield" of the source. Adding this solution of nonlinear Maxwell's equations and its Charge-Time inverse (absorption eigenfield or "advanced field"), the charges making the source cancel, so that we obtain a solution of the linear Maxwell's equations. The advanced field is a field converging to the virtual source; used to replace a real source in a mathematical representation, it has generally no physical existence, while approximate realisations may be tried[1].

Most usual sources (electron, atoms, molecules) are small in comparison with their distances, so that the electromagnetic field that they radiate is large close to them, small close to other sources. Using the trick, the absorption of the field radiated by a source, close to the source, requires the sum of a lot of small fields radiated by other sources, so that it requires a lot of time; meanwhile it remains a field. As this may be applied to all sources, it exists a residual field which is, at least, the field which remains in the dark, at 0 Kelvin. It is an absolutely normal field.

Far from sources, this field is stochastic, but it may be amplified by sources, leaving its pure stochastic nature[2]. From the building of electromagnetic fields by amplification of existing fields, it appears that it is physically meaningless to split the field $\mathbf{E}$ of a mode into a zero point part $\mathbf{Z}$ and the remainder: $\mathbf{Z}$ can only be obtained by an attenuation of $\mathbf{E}$ by an absorber, so that $\mathbf{E}-\mathbf{Z}$ is a difference of fields existing in two different points. The flux of energy available in an absorber such as a photocell is proportional to the difference between the square of the incident field $\mathbf{E}$ and the square of the emergent field $\mathbf{Z}$. The first approximation which neglects the zero point field fails in the detection of low fields, in particular in photon counting experiments.

The mean value of the energy of a field in a monochromatic mode is given by the second Planck's law:

$$W_\nu = h\nu\Big[\frac{1}{\exp(h\nu/kT)-1}+\frac{1}{2}\Big] \tag{4}$$

It is a mean energy, subject to fluctuations.

[1] In laser fusion experiments for instance.

[2] For this reason, we prefer "zero point field" to "stochastic field".

4 Excitation of a source: spontaneous and stimulated emissions.

The spontaneous emission is considered as an amplification of the energy of a mode, an absorption as a reduction of this energy, possibly down to the zero point energy. If the source is a molecule whose classical eigenstates are minimums of potential, a transition is possible if the molecule gets enough energy to reach the threshold between two minimums by an absorption, at Planck's frequency. If the threshold is very low, as in a photocell, the fluctuations of the zero point field are sufficient to provide the needed energy.

In classical electromagnetism, the absorption of an electromagnetic field is its cancellation by an opposite field; using Schwarzschild-Fokker's trick, a source is replaced by a field and the cancellation of fields may be studied at any time.

All electromagnetic fields may be decomposed on a set of orthogonal modes including the eigenmode of the source. All modes orthogonal to the eigenmode cannot exchange energy with the source (replaced by the advanced field), so that the source can only be excited by the component of fields on this mode.

It is physically impossible to build a "spherical"[3] wave to excite a point source. The exciting waves are, in the practice, nearly plane waves. The fraction of the wave which corresponds to the projection of the vector representing the eigenmode of the source on the vector representing the plane wave is absorbed; the remainder is scattered. The decomposition of the field radiated by the multipolar sources into plane waves is well known; one of the plane waves of the decomposition subtracts from the plane exciting wave, the other provide scattered waves. In the general case, the scattered fraction is low if the source has the properties of a good antenna.

The trick cannot be used to compute the energy, the energy must be deduced from all fields; this deduction is generally easy, at least at infinity, in a Fraunhofer computation: Set a_iE $(i = 0, 1, \dots)$ the amplitudes of the orthogonal plane waves resulting from the decomposition of the field radiated by a source excited by a wave of amplitude E, and a_0 the coefficient corresponding to the mode of the exciting wave. The input energy is proportional to E^2, the output to $E^2(|(1+a_0)|^2 + \Sigma_{i>0}|a_i|^2)$. The scattered energy is $W_S \propto E^2\Sigma_{i>0}|a_i|^2$, while the total absorbed energy is $W_T \propto E^2(2|a_0|\cos\phi + |a_0|^2)$ where ϕ is the difference of the phases between the exciting wave and the wave scattered in the same mode; the energy effectively absorbed by the source, which provides its excitation is $W_E = W_T - W_S$.

The ratio $\rho = W_E/W_T$ is between 0 and 1, the last case corresponding to a well adapted antenna.

Studying the spontaneous emission, it is generally obtained that the zero point field is two times more effective than an "ordinary field". The reason is that this ordinary field is the increase of the zero point field in a plane mode

[3]We use improperly "spherical" for the field emitted or absorbed by a point source such as a dipole while it is not invariant by all rotations around an axis which crosses the source.

amplified by the exciting source. Remark that the factor 2 requires a model of source, and the introduction the radiation reaction; the hypothesis are the object of a lot of discussions [5].

5 Conclusion.

The present computation of the spontaneous emission as an amplification of the zero point field does not apply to quantum electrodynamics which requires a free change of the mode of a photon through the "reduction of the wave packet", to allow, for instance, an EPR experiment.

The field which excites a molecule, stimulating an emission, is in a mode that the molecule is able to absorb, that is in the eigenmode of absorption for the transition. The plane mode generally used to excite a molecule, must be projected on the eigenmode while the regular zero point field, which is in the eigenmode of absorption, does not require a projection.

Thus the classical theory explains that an ordinary field is less effective than the zero point field to stimulate an emission, while quantum electrodynamics does not.

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