Can Astronomers Observe a Difference between a Doppler Effect and Coherent Parametric Raman Scattering?

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In a very low pressure gas, the Raman shift is not only spatially coherent, but parametric too. Spatial coherence means that the wave surfaces are preserved; parametric means that the shift is produced by all molecules of the gas, so that the frequency shift is not quantized, the same for all frequencies (except for a small dispersion), and there is no diffusion of the lines of a shifted spectrum. Thus only a small dispersion, similar to the dispersion in the refraction can differentiate Doppler effect from this scattering.

Introduction

In a previous paper [Moret-Bailly (1996)], we have shown that Raman scattering in a very low-pressure gas is coherent, so that the colour of an image is changed without any blur. But problems remained which prevented us from saying that the entire shift observed in galaxies is a Raman shift: in the usual observation, a Raman shift corresponds to a transitions between two levels so that the shift of frequency is quantized; to make this quantized disappear, it is necessary to consider very small (unobservable) Raman shifts. If a lot of such small shifts are added by a transmission of a light beam through a thick gas, a large shift may be obtained, but it is a diffusion of the shifts so that if there is a spectrum in the light beam, the lines fade.

There are two types of interactions between light and matter:
1. the first, used in photoductive cells, is a quantum effect, it was long the best known because it dominates when monochromatic light is used.
2. the second is the parametric effect, without transitions in the matter. An example in common use is refraction. Many other effects have been demonstrated by the use of very short pulses.

Our aim is to adjust the scales of time in a very low pressure gas, and, thus extend the theory set for short pulses in medium pressure gases to moderately coherent light, in very low pressure gases.
Quantum and parametric interactions

The classical theory of the interaction between light and matter gives the easiest way to understand the problem.

Assume that a molecule is a non-linear system which permanently exchanges energy with the surrounding electromagnetic field; in the dark, this electromagnetic field is the zero point field which is constant to a first approximation. Thus the molecule is a non-conservative system which possesses attractors called eigenstates in quantum mechanics; remark that these attractors are not perfectly well defined because the zero point field is stochastic. Assume that other electromagnetic fields interact through dipole moment (for instance) allowing jumps from one attractor (eigenstate) to another; the key of the classical quantification is to observe that the molecule remains near an attractor until a sufficient electromagnetic pulse causes it to jump to another attractor; a model of the molecule is obtained, assuming that the amplitude of the dipole moment is a function of the molecular variables which takes a zero value for the attractors; this behaviour of the dipole introduces a non-linearity of the set of molecules. This model works, though it is not perfectly realistic because it artificially splits the electromagnetic field into two parts, a constant part to define the attractors, and a useful part, sum of the fluctuations and the field sent by an experimental set into the system.

If an electromagnetic pulse hits the gas, all molecules are excited because the attractors are not perfectly well defined, so that we write that the molecules can only be near the attractors. Then, the non-linearity induces an electromagnetic concentration of the energy in a small number of molecules which reach another attractor [Monnot (1994)].

Lamb (1971) distinguishes two types of interaction of a medium: usual quantum interactions and parametric ones with ultra short light pulses. He writes that "ultra short" means shorter than all relaxations. He considered that the main relaxations are collisional; the classical model shows that there exists a relaxation time corresponding to the duration of the process of concentration of the energy necessary to obtain a transition. However the introduction of this second relaxation time is not important because both relaxation times decrease with the pressure.

Parametric interactions and coherence

Natural light may be considered as made out of pulses; the length of these pulses defines the time coherence of the light beam; Fourier transformation shows that to observe sharp absorption or emission lines, the pulses must be long enough, that is the time coherence must be sufficient.

To remain in the conditions of parametric interactions, the slower the relaxations, the longer the pulses may be. Thus, a light beam which carries a good spectrum may have parametric interactions if the relaxations are very slow, i.e. in a very low pressure gas. But, an experiment in the laboratory becomes impossible, because the lower pressure of gas, the longer the effective length of the gas cell must be. Luckily, there are very large regions in the universe where the gas pressure is extremely low.
In a parametric interaction, all molecules set on a same wave surface are excited and radiate simultaneously; if their number is large enough, they produce a wave which has exactly the same shape as the initial wave, so that an image is exactly reproduced, without any blur; it is the spatial coherence. We know that for the usual Raman effect or for the Rayleigh scattering, the spatial coherence is destroyed by collisions or by density fluctuations. Refraction is a very particular case of parametric scattering.

**Parametric Raman scattering with close energy levels**

Consider the interaction of a beam of frequency $\nu$ with a molecule which possesses two low-energy levels separated by $\hbar \mu$. The incident electric field is $E \cos 2\pi \nu t$ and the Raman Stokes scattered field $E \cos(2\pi(\nu-\mu)t)$; the sum of these two fields is:

\[
D = E \cos(2\pi \nu t) + E \cos(2\pi(\nu-\mu)t)
= E \cos(2\pi \nu t) + E \cos(2\pi \mu t) \cos(2\pi \nu t) + \sin(2\pi \mu t) \sin(2\pi \nu t) \tag{1}
\]

The Raman field scattered by a single molecule is much lower than the incident field, so that $a$ is much lower than 1; suppose that the length of the light pulse is less than $1/(2\pi \mu)$; thus, we may develop the corresponding trigonometric functions:

\[
D \sim E \cos(2\pi \nu t) + a + 2E \mu t \sin(2\pi \nu t)
- 2E \mu t \cos(2\pi \nu t) - \frac{4}{3} E \mu t \sin(2\pi \nu t) \tag{2}
\]

put

\[
\tan(\mu t) = \frac{2\pi \mu t}{1 + a} \tag{3}
\]

\[
D \sim E + a \cos(2\pi \nu t) + \sin(2\pi \nu t)
- 2E \mu t \cos(2\pi \nu t) - \frac{4}{3} E \mu t \sin(2\pi \nu t) \tag{4}
\]

The denominator $\cos(\mu t)$ is nearly 1; the second line is quadratic in $\mu$ and linear in $a$; it may be neglected because in the first line $\mu t$ is linear in both $a$ and $\mu$:

\[
D \sim E + a \cos(2\pi \nu t - \phi(t)) \tag{5}
\]

No Raman line appears, and the main line is slightly shifted because the frequency shift is distributed among all molecules; it is extremely small and may be considered infinitesimal.

The contribution of other levels and Stokes lines must be added. Integrating for a volume of gas, a large shift may be obtained, without broadening of the lines.

**Order of magnitude**

Suppose that two low lying levels have exactly the same properties, in particular of symmetry; thus, the relative amplitude $A$ scattered by a thickness $e$ of gas is the same for the four possible scattering processes (two for refraction, Stokes and anti-Stokes); if the temperature $T$ of the gas is low enough, the difference of populations of the levels $N_1 - N_2$ is proportional to $\hbar \mu/kT$. The refractive index is:
n = 1 + \frac{2cA}{2\pi ve}

The proportion of atoms whose shift is not compensated by the opposite shift is:

\[ \frac{N_1 - N_2}{N_2 + N_1} = \frac{\Delta n}{2kT} \]

and the final shift is

\[ \Delta \nu = -\frac{A \mu_n^2}{2kT} = -\frac{\pi ve^2 - 1}{2ckT} \]

If the dispersion is not too large, \( \Delta \nu \) is proportional to \( v \), so that the effect may be confused with a Doppler shift.

If the redshift is not too large, by Hubble's law:

\[ \frac{\Delta \nu}{\nu L} = \frac{1}{L} \sqrt{c - \frac{LH_0}{c}} \approx -\frac{H_0}{c} \]

Using (8), (9), and the dispersion formula:

\[ n - 1 = \frac{2kTH_0}{\pi \mu_n^2} = \frac{N e^2}{8\pi^2 me_0 \mu_0} \]

where \( \mu_n \) is UV frequency, \( e \) and \( m \) the charge and mass of an electron.

The number of scattering molecules per cubic meter is:

\[ N = \frac{16\pi e_0 m_k}{\hbar^2} \left( \frac{k}{k + 2} \right)^2 \frac{1}{H_0 T} = 3.3 \times 10^8 \left( \frac{1}{H_0 T} \right) \]

For a Hubble constant \( 2.5 \times 10^{-18} \), an intergalactic temperature \( T = 2.7 \) K, and an average ratio \( \mu_0/m = 10^5 \), \( N = 22 \).

**Application to the spectroscopy of quasars**

In intergalactic space, at least in the halos of galaxies, molecules such as \( H_3^+ \) may have a convenient spectrum and a sufficient pressure to explain the redshift.

The energy lost by the redshift may contribute to the 2.7 K radiation, either by a quantum de-excitation of the molecules, or by an amplification of the existing radiation. Thus, isotropy is reached.

In the spectrum of a quasar, the same Ly\( \alpha \) line may appear in various places; their origin may be thin clouds falling with speeds of the order of magnitude of \( c \). But "the confinement mechanism of Ly\( \alpha \) clouds is still not known"[Shull (1995)].

In the atmosphere of a quasar, the magnetic (or electric) field may induce Zeeman (or Stark) structures; suppose that the splitting of a ground state is linear in the modulus of the field \( B \). The shift is proportional to \( B^2 \). If \( e \) is the mass of gas per unit of surface, the shift is

\[ d\nu = A \hat{\nu}, eB^2 \hat{\nu} e de, \]

where \( A(v,e) \) depends on the local nature of the gas. The variation of flux per unit of surface \( F(v,e) \) is given by an absorption coefficient \( G(v,e) \):

\[ df \hat{\nu}, e_1 = F \hat{\nu}, e_1 \hat{\nu}, e_1 de. \]
If the properties of the atmosphere are known, this formula (12) may be integrated numerically, to get $v$ as a function of $e$, then equation (13) may be integrated. Let us see what happens if $B = 0$ at a certain level.

Set $e_0$, a value of $e$ for which $B = 0$, and $m_n = e_0 - e$. For $e$ near $e_0$, we develop $B$ to the first order; thus $B$ is proportional to $m_n$, so that equation (12) becomes:

$$dv = A \cdot m_n^2 \cdot dm_n.$$  \hspace{1cm} (14)

$a_n$ is nearly constant for $m_n$ small; we may integrate:

$$v - v_n = \frac{a_n m_n^3}{3}.$$  \hspace{1cm} (15)

Relation (12) must be preserved to get the relation between $v_n$ and the frequency observed in the spectrum.

To simplify, suppose that a sharp line produces a constant absorption in $(v_0 - \epsilon/2, v_0 + \epsilon/2)$. The atom absorbs between the levels which correspond to the values of $e$:

$$e'' = -\frac{\epsilon}{2}, \quad e''' = \frac{\epsilon}{2}.$$  \hspace{1cm} (16)

If the saturation of absorption is negligible, the absorption is proportional to the length $L(v_n) = e'''' - e'''$, maximum for $v_n = v_0$. A study of $L(v_n)$ shows that the half width of the line is $1.01 \epsilon$. For each value $n$, a solution of equation $v_0 = v_n$, a Ly$\alpha$ line has an image in the spectrum.

**Conclusion**

The explanation of redshift of a continuous wave is only possible by Doppler effect, because, as the number of periods emitted and received during a certain time is not the same, the number of wavelengths which measure the length between the source and the receiver is not constant.

But, if the light is made out of pulses, other solutions work.

The effect described here, which could be called “redshifting refraction” is not a new effect, but the result of natural extrapolations of well known properties of the interaction between light and matter. Thus, it exists and must not be neglected without a demonstration that it is too small in the particular case considered.

**References**


