

# “Anomalous” frequency shifts and periodicities in astrophysics

## I. SPECTROSCOPY

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### I. 1 Conditions for Doppler-like frequency shifts.

Several authors searched “Doppler-like” frequency shifts produced by light-matter interactions to avoid an expansion of the Universe, but their solutions did not verify the following conditions:

#### I. 1. 1 Regular physics.

The introduction of new physical concepts must be justified by either a direct demonstration, or by a lot of explanations. It is better to use only old, reliable physics.

#### I. 1. 2 No blurring of the images: Coherent light-matter interactions:

**Same interaction** between any involved (molecular,...) dipole and the local involved electromagnetic fields.

#### Consequence:

From Huygens construction and Fresnel rules, if the number of involved molecules is large, the wave surfaces and images are clean.

#### I. 1. 3 No blurring of the spectra.

**A monochromatic wave must be transformed into single, frequency-shifted monochromatic wave.**

#### If the interactions are scatterings:

- i) the scattered wave must interfere with the exciting wave into a single frequency wave. (from the coherence, these waves have the same wave surfaces)
- ii) As the number of involved molecules is large, the individual exchanges of energy are infinitesimal, not quantified.

The molecules are perturbed by the waves to **non-stationary states** and return to the initial stationary state : it must be a **parametric effect** (i. e. **matter is a catalyst** allowing interactions of the waves).

Several waves exchange energy while **the molecules are not (de)excited** permanently.

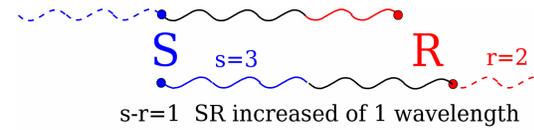
The exchanges of energy must obey thermodynamics: from hot beams, redshifted (usually light, from Planck's law) to cold beams (usually radiofrequencies).

#### I. 1. 4 (Nearly) constant relative frequency shifts.

A strict constant relative frequency shift  $\Delta v/v$ , as in a Doppler effect, is not required:

Observed variations of  $\Delta v/v$  are interpreted in the “big bang” theory by a variation of the fine structure constant (Webb et al. 1999) or by a variation of the ratio of the masses of proton and electron..

### I. 1. 5 Not Doppler frequency shifts.



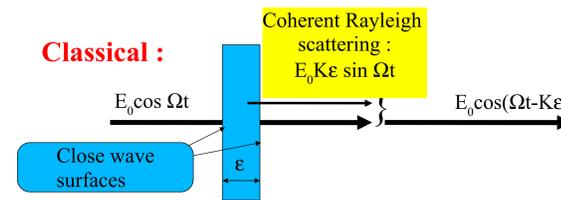
A *continuous wave* emitted by S is received at a lower frequency by R. The number s-r of cycles (wavelengths) between S and R increases : it is a Doppler effect.

**Consequence:** The theory of a Doppler-like effect must fail using a CW source.  
**Corollary :** **A time-coherence parameter must appear in the theory of a Doppler-like effect.**

### I. 2 Coherent Raman Effect on Incoherent Light (CREIL).

The refraction is a light-matter interaction which obeys the previous conditions, ... but does not change the frequencies ! Is it possible to replace the coherent Rayleigh scattering which produces the refraction by a coherent Raman scattering ?

#### I. 2. 1 Recall of refraction, that is of coherent Rayleigh scattering.



The thin sheet of matter lying between two close wave surface emits a coherent wave, having the same wave surfaces and same frequency than the exciting wave, delayed of  $\pi/2$ .

$$E = E_0 [\cos(\Omega t) + K\epsilon \sin(\Omega t)]$$

The amplitude remains  $E_0$  because the infinitesimal scattered wave is in quadrature: the energy is preserved.

$$E = E_0 [\cos(\Omega t) \cos(K\epsilon) + \sin(K\epsilon) \sin(\Omega t)] = E_0 \cos(\Omega t - K\epsilon) \quad (1)$$

Definition of the index of refraction n, setting:

$$K = 2\pi n/\lambda = \Omega n/c \quad (2)$$

#### Quantum point of view on refraction:

Set  $\Psi$  the (stationary) wave function of a refracting medium.

A perturbation by an electromagnetic wave  $W_i$  transforms  $\Psi$  into a “dressed” (non stationary) state  $\Psi_i = \Psi + \phi_i$ .

$\Psi_i$  emits the scattered wave delayed of  $\pi/2$ , having the same wave surfaces than the exciting wave.

Two simultaneously refracted EM waves  $W_i$  and  $W_j$  transform  $\Psi$  into  $\Psi + \phi_{ij}$ .  $\phi_{ij}$  is not equal to  $\phi_i + \phi_j$  if it exists an interaction operator O such that  $(\phi_i | O | \phi_j) \neq 0$ .

O transfers energy, produces frequency shifts of the refracted beams.

O may result from a global behaviour (plasma) or from molecular properties, through Raman type interactions because the molecular states excited by  $W_i$  and  $W_j$  have the same symmetries.

**This suggests to look at Coherent Raman Effects !**

### I. 2. 2 Replacing coherent Rayleigh scattering by coherent Raman.

At the start of a light pulse, the Raman scattered waves are in phase, so that, to conserve the energy, the amplitude of the exciting wave must be decreased.

**FOR COHERENT ANTI-STOKES SCATTERING**, (1) is replaced by:

$$E = E_0 [\sin(\Omega t)(1 - K'\epsilon) + K'\epsilon \sin((\Omega + \omega)t)] \quad \text{with } K' > 0$$

$$= E_0 [\sin(\Omega t)(1 - K'\epsilon) + K'\epsilon \sin(\Omega t)\cos(\omega t) + K'\epsilon \sin(\omega t)\cos(\Omega t)]$$

$K'\epsilon$  is infinitesimal, and we must assume that  $\omega t$  is small to avoid a possible separation of both frequencies by detection of beats :

$$E \cong E_0 [\sin(\Omega t) + \sin(K'\epsilon \omega t)\cos(\Omega t)]$$

$$\cong E_0 [\cos(K'\epsilon \omega t)\sin(\Omega t) + \sin(K'\epsilon \omega t)\cos(\Omega t)] = E_0 \sin[(\Omega + K'\epsilon \omega)t] \quad (3)$$

**FOR STOKES SCATTERING**,  $K'$  is replaced by a negative  $K''$ .  $K'+K''$  is proportional to  $\exp(-h\omega/2\pi kT) - 1$ , approximately to  $\omega/T$ . Thus, the frequency shift is proportional to

$$\Delta\Omega = (K'+K'')\epsilon\omega \propto \epsilon\Omega\omega^2/T. \quad (4)$$

As in refraction, the  $K$ s are proportional to  $\Omega$  if the dispersions of the polarisabilities are neglected. Thus  $\Delta\Omega/\Omega$  is nearly constant.

### I. 2. 3 Preservation of space-coherence with ordinary light :

#### Low frequency Raman resonance.

The interactions starting at the beginning of a pulse, keeping  $\omega t$  small along a light impulsion, requires a Raman period larger than the length of the impulsion, that is than the coherence time.

#### Low pressure gas.

To avoid a destruction of the space coherence, the collisional time must be longer than the coherence time.

We verify that “ultrashort” (i.e. “shorter than all relevant time constants”) light pulses allow to keep the coherence. (Lamb 1971, Yan et al. 5)

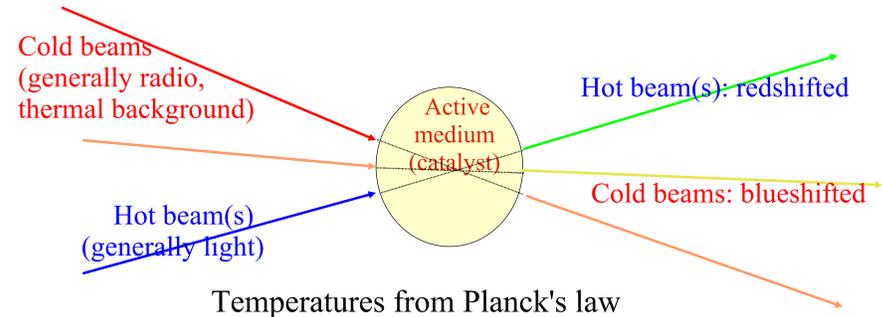
#### A problem :

It cannot be any permanent exchange of energy between the molecules and the light because **the molecules must return to their stationary state after the interaction** ... It is necessary that the interaction involves several beams, so that the final balance of energy is zero for the molecules.

This coherent effect is “**parametric**”: The molecules act as a catalyst.

The “**Coherent Raman Effect on Incoherent Light**” (CREIL) is a **SET** of elementary coherent Raman effects (followed by interference with the exciting beams) in which all transfers of energy obey thermodynamics .

## CREIL effect

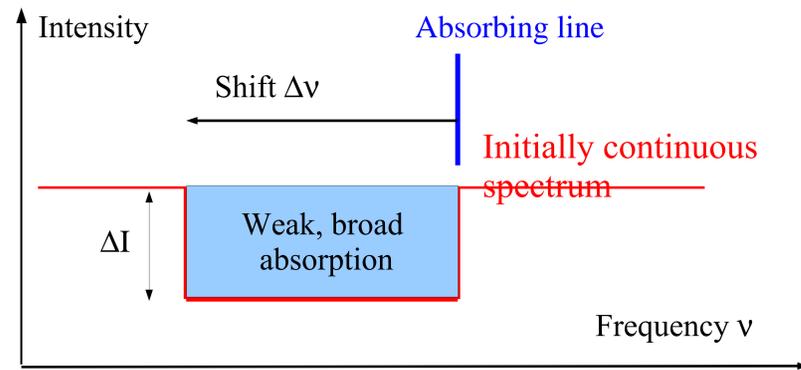


Thermodynamically allowed transfers of energy  
 No blurring of images (space-coherence preserves the wave surfaces)  
 No blurring of the spectra  
 Nearly constant relative frequency shifts  $\Delta\nu/\nu$

## II PROPAGATION OF A FAR UV CONTINUOUS SPECTRUM BEAM IN ATOMIC HYDROGEN.

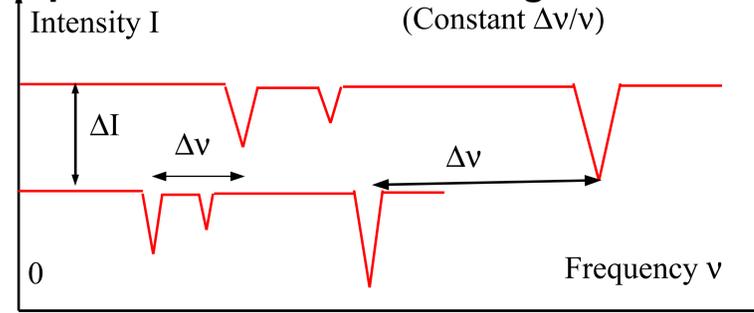
The light excites the  $Ly_{\alpha}$  line, populating the 2P level, so that a CREIL effect appears.

### II. 1 Invisible new lines,



The linewidth of a sharp line becomes equal to the frequency shift which may be large. The resulting broad, weak lines are usually mixed, they cannot be observed.

### ... improved contrast of existing lines.



The column density of  $H^*$  which needed to produce the redshift  $\Delta\nu$  depends on the initial state of the gas and the absorbed intensity  $\Delta I$ , but not on the initial intensity, so that the contrast of the lines is improved.

## II. 2 Propagation of a far UV continuous spectrum in atomic Hydrogen.

### II. 2.1 Multiplication of the lines (look at the following figure)

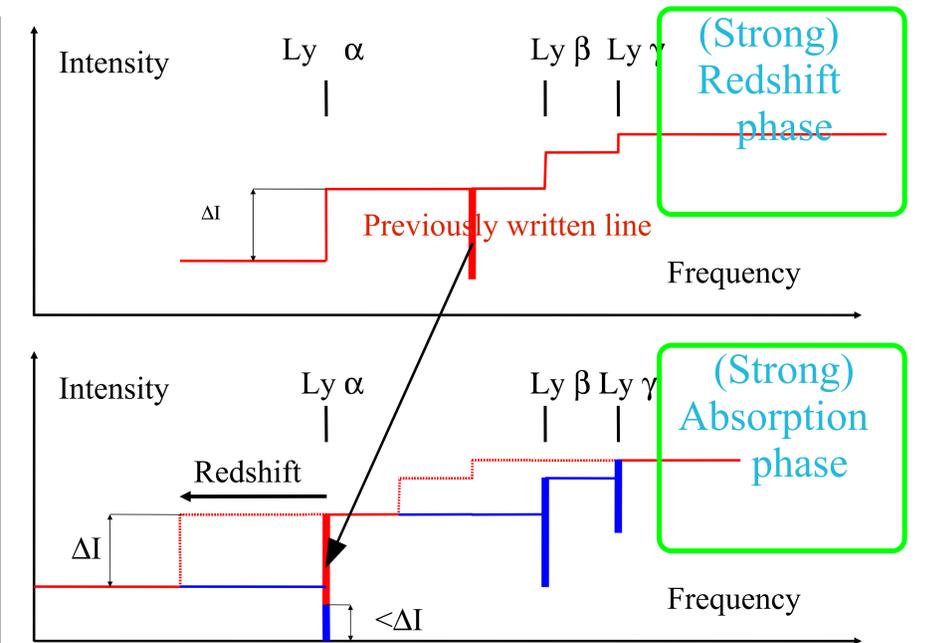
During the strong redshift phase, the intensity at the Lyman  $\alpha$  frequency is larger than the intensity  $\Delta I$  whose absorption is needed for a permanent redshift. Therefore each gas line absorbs (or emits) a constant intensity which is usually too low to be observed.

The strong absorption phase happens when an absorbed line gets the Lyman  $\alpha$  frequency:

the redshift *almost* stops, so that all gas lines are strongly absorbed (or emitted).

*“Almost” because higher states are excited, which produce weak redshifts, and from which the atoms may decay to the 2S or 2P states.*

Thus, a redshift phase restarts, until, at least, the frequencies of the  $\beta$  and  $\gamma$  lines which were just written get the  $\alpha$  frequency.



All lines of the gas are absorbed when an absorbed line is redshifted to the Lyman  $\alpha$  line, in particular Lyman  $\beta$  and  $\gamma$ .

### II. 2.2 Karlsson periodicities

The absorbed Lyman  $\beta$  and  $\gamma$  are shifted to the  $\alpha$  by a frequency shift of the light relative to the  $\alpha$  frequency, equal to:

$$Z = (\nu_{\beta} - \nu_{\alpha}) / \nu_{\alpha} = 3 * 0.062 = 3 * Z_b$$

$$Z = (\nu_{\gamma} - \nu_{\alpha}) / \nu_{\alpha} = 4 * 0.062 = 4 * Z_b$$

The parameter  $Z_b$  is found in the spectra of strongly redshifted objects

(quasars); thus their redshifts appear strongly connected to atomic hydrogen and a source of UV light.

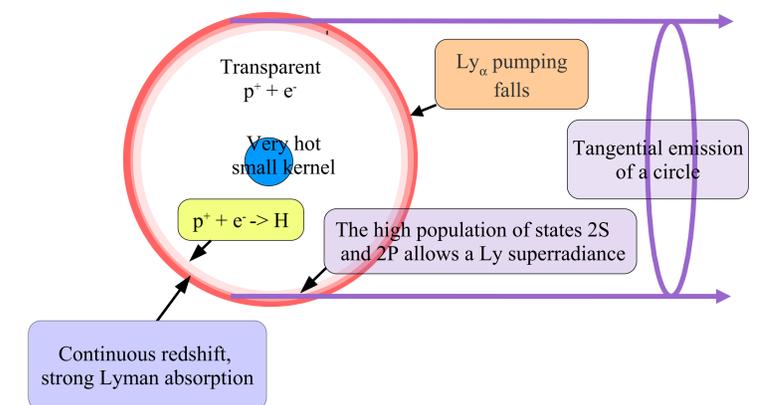
The patterns resulting from this process are redshifted by following interactions, so that, while the relative redshifts are constant, the finally observed shifts are reduced. A consequence is a logarithmic law for the observed redshifts.

## II. 3 Structuring the redshifting gas.

With a single, small source of UV light, the redshift and absorption phases correspond to spherical shells of gas. The pumpings of several sources add, so that the shells are more complicated.

### Generating circles.

A low pressure of the gas reduces the absorptions by impurities, so that the periodicities do not appear.



### I. 2.4 Application to astrophysics.

Lamb's conditions are fulfilled in any matter using femtosecond pulses; the coherence time of ordinary light is much longer (some nanoseconds).

Long enough a collisional time requires a gas pressure generally lower than 1000 pascals.

The frequency of the required Raman resonance must be lower than some MHz, but not too low (formula 4 : shift proportional to  $\omega^2$ ) to produce strong CREIL effects. Such low frequencies are not common, or they appear in states whose population is low.

**Neutral atomic hydrogen** has a too high Raman frequency (1420 MHz) in its ground state. It has convenient frequencies **in the  $2S_{1/2}$  states (178 MHz), in the  $2P_{1/2}$  states (59 MHz) and in the  $2P_{3/2}$  states (24 MHz).**

**Hydrogen in these states will be noted  $H^*$ .**

### I. 2.5 Quest for “anomalous” frequency shifts.

**It is a quest for excited atomic hydrogen  $H^*$  or similar molecules.**

As the CREIL effect increases the entropy of a set of simultaneously refracted beams, we must search the temperature of these beams by Planck's law. Usually, the high frequency beams are hot, therefore redshifted; the thermal, radio beams are cold, blueshifted.

$H^*$  is mainly obtained:

Thermally around 100 000 K if a sufficient pressure forbids an ionization.

**Around 15 000 K, with a Lyman $\alpha$  pumping.**

By a cooling of a plasma.

# III APPLICATIONS TO ASTROPHYSICS

## III. 1 In the solar system

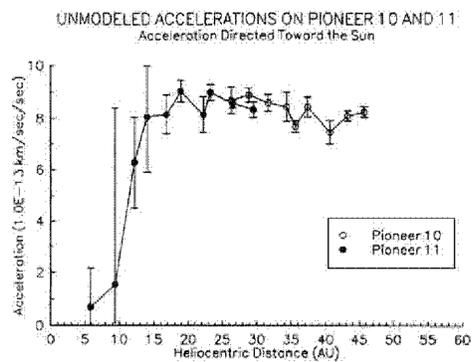
### III. 1. 1 Transfer of energy from solar light to radio frequencies.

Beyond 5 UA, the cooling of the solar produces atomic hydrogen in excited states; in particular, the 2S state is metastable at low pressure. In this hydrogen, energy is transferred from the solar wind to radio, thermal waves, producing blueshifts.

③ In particular:

#### - Anomalous accelerations

After corrections of the frequency shifts produced via the Doppler effect by known accelerations (Sun, planets, Kuiper asteroids, ...), the cosmic microwave background remains blueshifted, that is heated (Anderson 2002, Markwardt 2002). As the corona which generates the wind has an anisotropy bound to the ecliptic, the fluctuations of the CMB are, in particular, bound to the ecliptic:



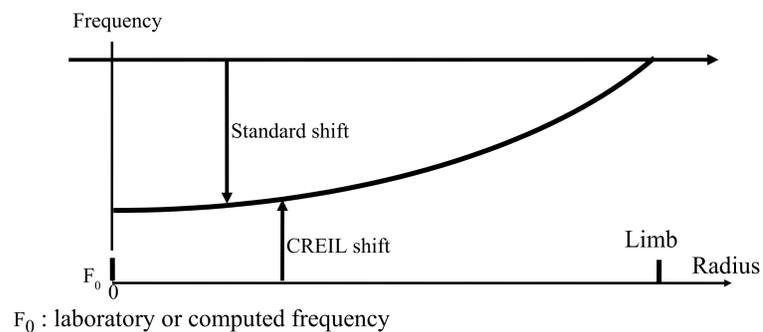
This frequency shift is possible for the CW radio wave because the sunlight is incoherent and the radio wave strongly mixed with noise. The standard explanation by a Doppler effect and an anomalous acceleration of the probes toward the Sun requires a change of the celestial mechanics.

#### -Binding of the anisotropy of the CMB to the ecliptic.

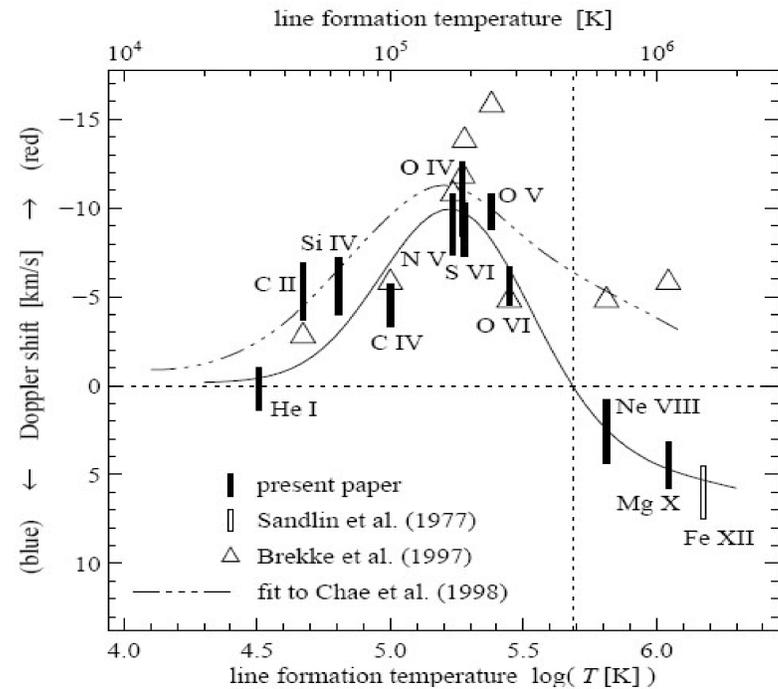
The development of the CMB into spherical harmonics show not only the movement of the Sun to the apex, but, by the quadrupoles and octupoles, a binding to the ecliptic. (Land & Magueijo 2005), Naselsky et al. 2005, Schwarz et al. 2004) As the solar wind is generated by the corona whose anisotropy is bound to the ecliptic, the explanation by a CREIL effect is evident.

### III. 1. 2 Frequency shifts of the far UV lines of the Sun.

After corrections due to the movement of SOHO in particular, the shift of these lines is a function of the radius of the solar disc. Peter & Judge (1999) use the standard interpretation of the redshifts by spicules or siphon flows, so that the shifts are supposed zero at the limb, in contradiction with the lab or theoretical determinations. Accepting old frequencies, the shifts whose directions are opposite are non-zero at the centre and large at the limb of the Sun:

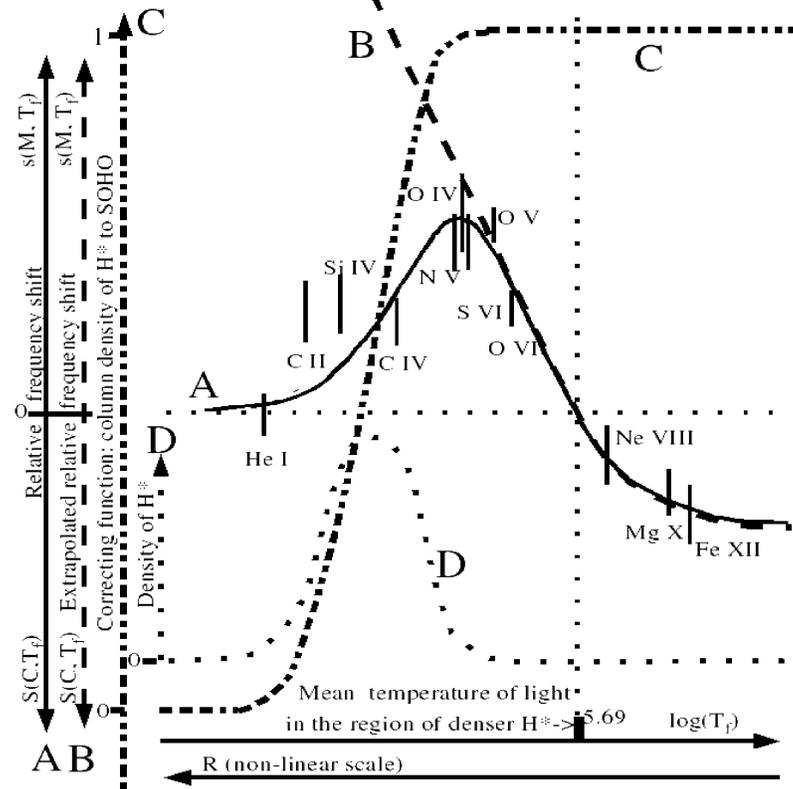


Observing the lines at a point of the Sun, Peter & Judge found that the frequency shift is clearly related to the temperature at which the lines are emitted:



Usually, the energy flows from hot to cold, so that the signs chosen by Peter & Judge for the redshifts do not seem convenient. With our signs, the right, decreasing part of the curve obeys more thermodynamics, supposing that the lines exchange energy during a propagation in excited atomic hydrogen, to the mean temperature corresponding to the emission of He I.

Supposing that hydrogen cannot ionize, it contains a high concentration of  $H^*$  around 100 000 K ( $\log(T)=5$ ). On the following figure, curve D represents the concentration of  $H^*$ , and C, the integral of D, the column density of  $H^*$  crossed by light propagating from right to left. Curve A built by Peter & Judge represents a function product of functions C and D.



#### Problems:

With the direction of propagation of the light that we have chosen, the temperature decreases with an increasing altitude  $R$  while, in the chromosphere from which it seems that the lines are emitted, it increases.

Can the previous result apply under the photosphere ?

Under the photosphere, hydrogen is neutral atomic or dissociated.

Atomic hydrogen is far from metallic state, so that the far UV energy is too low for a dissociation and too high for an absorption between atomic levels. Protons and electrons do not absorb: the gas is transparent.

Can the lines be sharp and the CREIL work at the high pressure ?

Yes, at very high pressures, the Galatry lineshape is sharper than the Doppler thermal lineshape in a low pressure gas.

Can the light cross the chromosphere ?

Not easily, mainly through the spots where the ion  $H^-$  is not abundant. The far UV lines are very intense close to these spots.

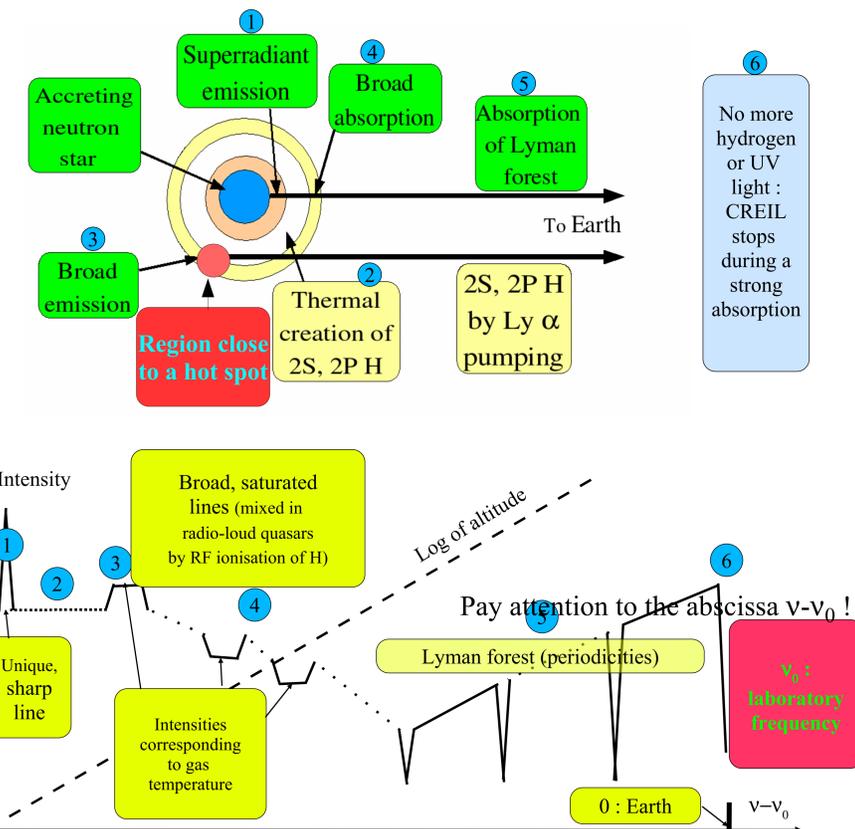
Can the photosphere emit far UV lines too?

Yes, by a Rayleigh incoherent scattering.

## III. 2 High redshift objects

### III. 2. 1. Spectrum of the quasars

#### Model of quasar, as an accreting micro-quasar



Using  $v-v_0$  in abscissa puts the emitted or absorbed by different transitions to the same place, so that the high energy emissions and low energy absorptions may be represented.

- 1 : Sharp line emitted in a hot region close to the kernel; No CREIL because hydrogen is fully ionized.
- 2 : At 100 000 K, with a pressure of the order of 10 000 Pa, hydrogen is not fully ionised, but partly excited to a principal quantum number 2 (H\* hydrogen). The resulting permanent redshift forbids the printing of visible lines, a gap appears in the spectrum.
- 3 : The *emitted* broad lines may be generated beyond the kernel (close to a hot spot), so that their redshift may be larger than the redshift of the sharp lines.
- 3,4 : Pressure of the order of 100-1000 Pa. The lines are broadened by saturation and a weak, simultaneous, thermal CREIL. If the quasar is “radio-loud”, the radio frequencies ionize hydrogen, so that there is no CREIL, no multiplication of the lines (Anderson et al. 1996). The lines result from a very broad emission followed by slightly sharper absorptions.
- 5 : Lyman forest, periodicities (Bell et al. 2002, 2003); the value  $\Delta I$  required to produce a permanent redshift decreases with the pressure because the collisions de-excite hydrogen less and perturb the CREIL effect less. Therefore, for a given redshift, the absorption is lower, the mean intensity of the spectrum larger.
- 6 : A consequence of the efficiency of the Lyman pumping is that there is a large probability that the process stops during a vanishing absorption.

The micro-quasars were found by astronomers who looked for quasar in the radio and X ranges. In these regions, their spectra are identical to the spectra of the quasars, but they are very weak in the visible, and they are fast moving in our galaxy. They seem correspond to “accreting neutron stars” which, following the theory of neutron stars should be bright, but are NEVER observed (Popov et al. 2003)

### III. 2. 2 Arp's systems of quasars and galaxy.

Arp (2003) found systems of several quasars and a single galaxy, in which the positions of the objects are remarkable, for instance on a line: these objects may be connected by a visible bridge of gas or dust. The redshift of the galaxy is usually much lower than the redshifts of the quasars, although it seems larger than the redshifts of similar galaxies

The UV radiated by the quasars produces H\*, so that these systems are surrounded by absorbing/redshifting shells of atomic hydrogen.

As hydrogen is more excited by the far UV radiation close to the quasars which emit it, there is more H\* on the path of the light from the quasars which, therefore, are more redshifted.

It appears that the quasars were ejected by the galaxies.

## III.3 Intergalactic structure.

### III. 3. 1 Relation between quasars and galaxies.

If there is more hydrogen around our galaxy than inside, the micro-quasars become “isolated quasars” when they leave it, their repartition is nearly isotropic. The other quasars are bound to their own galaxies.

The transformation of microquasars into quasars when they leave their galaxies shows that, at least around the galaxies, the density of hydrogen is larger than between the stars making the galaxies.

### III. 3. 2 The “Very Red Objects”.

The VROs (Hall et al. 2001, Wold et al. 2003, Boller et al. 2003) are generally close to the quasars : the far UV emitted by the quasars (or similar objects) creates

excited atomic hydrogen which produces a CREIL effect.

These objects appear surrounded by hot dust (up to 100K) whose stability is a problem. It is probably not dust, but the CREIL counterpart of the redshifts which loses a lot of energy.

Arp's observations, and more generally the proximity effect show that the size of the regions in which there is much low pressure hydrogen is usually larger than the size of the galaxies.

### III. 3. 3. Origin of Tiftt-Napier periodicities in galaxy spectra.

Periodicities 36 and 72 km/s are observed by a Fourier analysis and a statistical study of the spectra of the galaxies (Tiftt 1976, Burbidge 1968). Assuming that the origin of these periodicities is similar to the origin of the periodicities in the quasars, the role of the difference of energy between the Lyman lines seems plaid by the difference of energy of rotational lines (P or R branches) while the energy of the Lyman lines corresponds to the energy of an electronic transition, the ratio of these energies being an integer multiple of 36/300 000 or 72/300 000. The existence of two periodicities seems a result from an alternating intensity of the lines: depending on the intensity of the pumping, either all lines, or only the strongest produce the periodicities; the atoms of hydrogen having a spin  $\frac{1}{2}$ , molecule H<sub>2</sub> has a convenient periodicity.

The frequencies of the lines in the P and R branches of the (2,0) Lyman band of H<sub>2</sub> are 92 685, 92 839, 92 790 and 92 659 cm<sup>-1</sup>. A periodicity of the order of 25 cm<sup>-1</sup> appears, which corresponds to a Doppler shift of 81 km/s.

This value is larger than 72 km/s, but such a difference may be easily understood because the relative frequency shifts decrease by a logarithmic law (to take into account the shifts which follow the multiplication of the lines), and the value 72 km/s result from an averaging.

However, it remains to do a big spectroscopic work, taking into account the intensities of the lines at various temperatures, and studying the hyperfine structures !

The redshifting process of the galaxies works using the radiations of all stars of the galaxies, because light from all origins builds a single set of redshifting or absorbing shells.

The light coming from a galaxy gets two periodic intensity modulations, from the observed galaxy and from our galaxy. These modulations may be in phase or out of phase.

The modulation due to atomic hydrogen is so stronger than this modulation that, in the case of the quasars, the 36 km/s is not visible.

### III. 3. 4 The cosmic microwave background

The energy lost by the redshifts blueshifts, that is heats the thermal background. This heating is particularly large close to the bright, redshifted sources.

Within radiowaves, the CREIL effect is resonant, particularly large, leading to a thermal equilibrium, including isotropy.

The observation of the quasars and the galaxies shows that the regions where a CREIL active effect exists by an UV pumping of atomic or molecular hydrogen are wide: therefore, except for an observation close to bright objects, the thermal equilibrium is reached, the CMB is almost isotropic; only the movement of the solar system, and the amplification of the CMB by the cooled solar wind trouble the equilibrium.

## Conclusion:

**“Anomalous” frequency shifts of light beams are produced by their propagation in gases obeying Lamb's conditions.**

**In astrophysics, the main active gas are:**

**- Neutral atomic hydrogen in states 2S and (or) 2P mainly produced by a Lyman  $\alpha$  pumping which requires a very hot source, an accreting neutron star named quasar.**

**- Cold molecular hydrogen pumped by the light of the galaxies.**

**Taking the “Coherent Raman Effect on Incoherent Light” into account allows to explain a lot of observations from elementary spectroscopy and thermodynamics, without any need of dark things.**

**Try to explain the 36 km/s periodicity in the spectrum of the galaxies : it is probably a simple problem !**

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